1. (10 points) Find an approximate value of $\int_{2}^{1} x^{-2} \, dx$ using composite Simpson’s rule with $h = 0.25$. Give a bound on the error. Then calculate the exact value of the integration and compute the exact error to see if the error bound is accurate.

2. (10 points) Obtaining an upper bound on the absolute error when we compute $\int_{0}^{6} \sin x^2 \, dx$ by means of the composite trapezoid rule using 101 equally spaced points? (Be careful what is $n$ in your formula).

3. (10 points) Determine the coefficients $A_0$, $A_1$, and $A_2$ that make the formula

$$\int_{0}^{2} f(x) \, dx \approx A_0 f(0) + A_1 f(1) + A_2 f(2)$$

exact for all polynomials of degree 2 or less.

4. (10 points) Using the Romberg scheme, establish a numerical value for the approximation

$$\int_{0}^{1} e^{-(10x)^2} \, dx \approx R(1, 1)$$

Compute the approximate to only three decimal places of accuracy. (Hint: Compute $R(0, 0)$ and $R(1, 0)$ first, then compute $R(1, 1)$ based on $R(0, 0)$ and $R(1, 0)$).

5. (10 points) For a decreasing function $f(x)$ over an interval $[a, b]$ with $n$ uniform subintervals, show that the difference between the upper sum and the lower sum is given by

$$\frac{(b - a)}{n} [f(a) - f(b)].$$

6. (10 points) Compute two approximate values for

$$\int_{1}^{2} \frac{1}{x^2} \, dx$$

using $h = 1/4$ with lower sums and the composite trapezoid rule, and compare them with the exact solution and see which one computes more accurate solution.