Models of Computation **CS 575**
Spring 2009 **Sample Midterm Questions**

Note: this is much more than will be on the exam.

1. For each of the following sets, determine whether it is

   - Regular
   - Context-free and not regular
   - Turing recognizable and not context-free

   For each language, prove your answer.

   (a) $\{a^n b^{2n} a^n | \ n \in \mathrm{N}\}$
   (b) $\{x \in \{a,b\}^* | \$ the number of as in x or the number of bs in x is divisible by 3$\}$
   (c) $\{0^i 1^j 2^j | \ i,j \in \mathrm{N}$ and $i \geq 1\}$

2. True or False? Sketch a proof or give a counterexample:

   If L_1 is regular and L_2 is context free then $L_1 \cup L_2$ is context free.

   If L_1 is not regular and L_2 is not context free then $L_1 \cap L_2$ is not regular.

3. Draw a DFA that accepts the language given by the following regular expression: $(0 \cup 01)^* \cup (01 \cup 10)1^*$.

4. Draw a PDA that accepts the language $\{a^n b^n : n \geq 0\}$.

5. Give a regular expression for the language accepted by (insert an NFA here).

6. Draw a DFA that is equivalent to (insert an NFA here).

7. Give a context free grammar for the language $\{x^n y^n : n \geq 0, x, y \in \Sigma, x \neq y\}$.

8. Let $L = \{a^n b^n : n \geq 0\}$. What are the equivalence classes for the relation R_L? What does this tell you about L?

9. Give the state diagram for a Turing machine that recognizes $\{a^n b^k : n < k\}$.

10. A 2-D Turing machine (2-D-TM) has a tape consisting of cells $c_{i,j}$, $i,j = 0,1,2,\cdots$. There is a single tape head that can move up, down, left, or right (U, D, L, R) at each step. Eg, moving up means moving from $c_{i,j}$ to $c_{i,j+1}$. The input is initially stored in cells $c_{0,0}, c_{1,0}, \cdots, c_{n-1,0}$.

    Prove that every 2-D-TM can be simulated by an ordinary single tape DTM.