Logic and Formal Models **CS 375**
Fall 2009 **Practice for First Midterm**

For each problem, explain your answer or show how it was derived.

1. Prove by induction that 6 divides $n^3 - n$ for all $n \in \mathbf{N}$.

2. Suppose A and B are finite sets, and $f : A \to B$ is a one-to-one function that is not onto (injective but not surjective). What does that say about the sizes of A and B?

   Give a one-to-one, non-onto function from $\mathbf{N} \to \mathbf{N}$. What does that say about the size of $\mathbf{N}$?

   True or false: If $|A| = |B|$ and $f : A \to B$ is a one-one function, then f must be onto. (Justify your answer.)

3. For each of the following sentences of propositional logic, state whether they are tautologies (always true), satisfiable, or never satisfied.
   - $(p \to (q \vee r)) \to (p \to q)$
   - $((p \vee q) \to r) \to (p \to r)$
   - $((p \vee q) \wedge (\neg p \vee \neg q))$

4. For each of the following relations, say whether it is reflexive, whether it is transitive, whether it is symmetric.
   - $\{(1, 3)\}$
   - full siblings (as opposed to half-siblings), over the universe of people
   - $\{(n, m) \in \mathtt{N} \times \mathtt{N} : \ n + m \text{ is odd}\}$
   - $\{(n, m) \in \mathtt{N} \times \mathtt{N} : \ n \text{ divides } m\}$ (We say n divides m if $\frac{n}{m}$ is an integer.)

5. You are asked to prove that Mercedes eats Hildegard, given the following sentences. Your first task is to define the predicates you will use, and then to turn the English sentences into sentences of the predicate calculus. Then apply proof rules to reach the desired conclusion.

   (a) Hildegard is an iguana.
   (b) Mercedes is a jaguar.
   (c) Jaguars eat iguanas if they catch them.
   (d) Jaguars catch slow iguanas.
   (e) Fat iguanas are slow.
   (f) Hildegard has gained a lot of weight.

6.
   - Put the following formula, φ, into Prenex normal form: $\varphi = (\exists x \forall y (x \leq y)) \to \forall y \exists x (x \leq y))$.
   - Give $\neg\varphi$ in Prenex normal form.
   - Give a quantifier-free formula that is equivalent to φ.